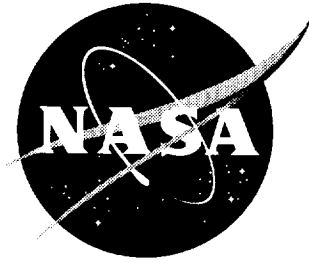


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# A Back Face Strain Compliance Expression for the Compact Tension Specimen

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The effect of a finite-sized back-face-mounted strain gage was determined by taking the difference in y-direction displacements at two symmetric points on the back face, and dividing by the gage length between the two points. The J-integrals were determined from the elasticity solutions [9]. These values were used to calculate the stress intensity factors.

## Results

Predicted stress intensity factor and crack mouth opening relationships are compared to corresponding relationships from the literature. Good agreement with these well established relationships increases the confidence in the numerical analyses, allowing a reliable relationship for back faced strain to be developed.

### Stress Intensity Factors

The FRANC2D-calculated and previously published [5,6] relationship between  $K$  and crack length for the compact tension specimen is shown in Figure 3. Here,  $K$  is normalized by  $BW^{1/2}/P$ , where  $B$  is the specimen thickness,  $W$  is defined in Figure 1, and  $P$  is the applied load. For the range in which the published solution is valid ( $a/W \geq 0.2$ ) the values agree within 0.3%. The FRANC2D-calculated stress intensity factor is 8.7% greater than that predicted by the published relationship for  $a/W = 0.1$ . However,  $a/W = 0.1$  is outside the stated range of validity for the equation, and in most cases, will be less than the initial notch length. It should be noted that there is good agreement between the FRANC2D-calculated and published value of stress intensity factor for  $a/W = 0.9$ , even though the ratio of ligament length to crack length is small enough that the validity of linear elastic fracture mechanics might be questioned.

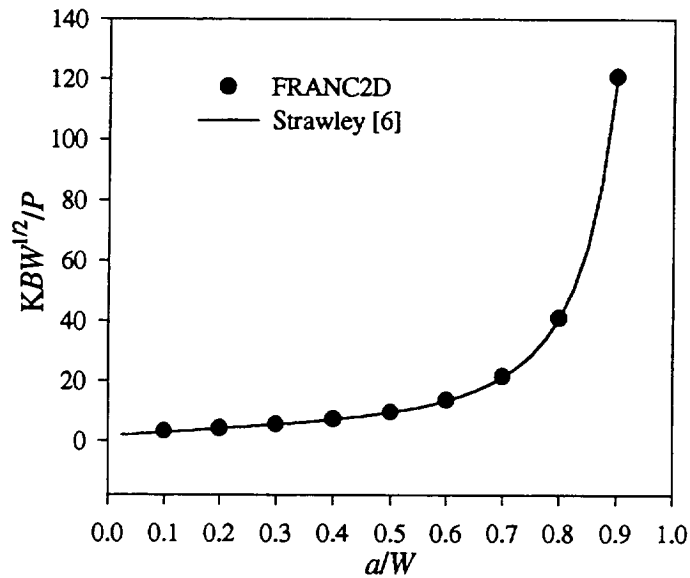


Figure 3. Dimensionless stress intensity factor versus  $a/W$ .

## Crack Mouth Opening

Saxena and Hudak [4] have presented an expression for crack length to width ratio ( $a/W$ ) as a function of crack mouth opening. Their expression is of the form

$$a/W = C_0 + C_1U + C_2U^2 + C_3U^3 + C_4U^4 + C_5U^5 \quad (1)$$

where the coefficients  $C_i$  depend on the specific clip gage location and  $U$  is a non-dimensional crack mouth opening, defined by

$$U = \left\{ [EvB/P]^{1/2} + 1 \right\}^{-1} \quad (2)$$

where  $E$  is Young's modulus,  $v$  is the crack mouth opening displacement,  $P$  is the pin load, and  $B$  is the thickness of the specimen. This solution is valid for  $0.2 \leq a/W \leq 0.975$ . The FRANC2D-calculated relationship is compared to the Saxena and Hudak solution in Figure 4. For a given  $a/W$ , FRANC2D-predicted crack mouth opening agrees well with the published solution.

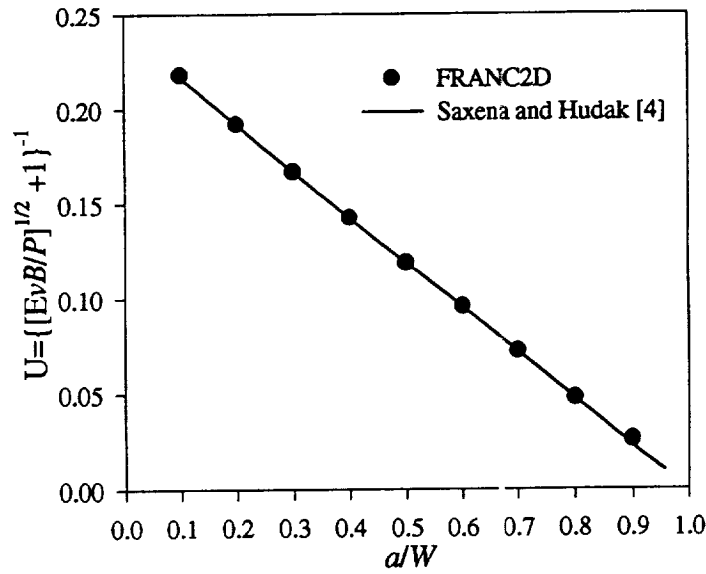


Figure 4. Crack mouth opening parameter ( $U$ ) versus  $a/W$ .

## Back Face Strain

The dimensionless back face strain parameter typically presented in the literature is

$$A^* = \epsilon EBW / P \quad (3)$$

where  $\epsilon$  is back face strain,  $E$  is Young's modulus,  $B$  is thickness,  $P$  is pin load, and  $W$  is defined in Figure 1. Results from the FRANC2D calculations and the previously published  $A^*$  versus  $a/W$  relationship are shown in Figure 5, and given in Table 1.

The relationship between  $A^*$  and  $a/W$  is fitted to a polynomial of the form

$$a/W = N_0 + N_1(\log A^*) + N_2(\log A^*)^2 + N_3(\log A^*)^3 + N_4(\log A^*)^4 + N_5(\log A^*)^5 \quad (4)$$

This is the form presented by Piascik, *et al.* for a back face strain expression for the extended compact tension specimen [10]. The compact tension specimen polynomial coefficients, based on the numerical calculations presented herein, are given in Table 2.

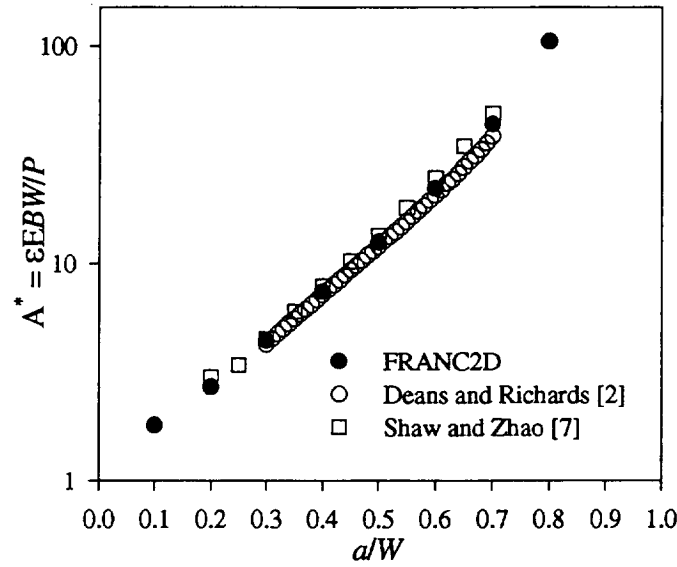


Figure 5. Back face strain parameter ( $A^*$ ) versus  $a/W$ .

Table 1--Back face strain parameter ( $A^*$ ) versus  $a/W$ .

$a/W$	$A^*$		
	FRANC2D	Deans and Richards [7]	Shaw and Zhao [8]
0.10	1.803	-	-
0.15	-	-	-
0.20	2.710	-	3.002
0.25	-	-	3.405
0.30	4.436	4.26	4.492
0.35	-	5.56	5.999
0.40	7.422	7.17	7.868
0.45	-	9.31	10.240
0.50	12.581	12.0	13.461
0.55	-	15.6	18.076
0.60	22.197	20.6	24.834
0.65	-	27.8	34.687
0.70	43.003	38.3	48.787
0.75	-	-	-
0.80	100.076	-	-
0.85	-	-	-
0.90	420.500	-	-

Table 2--Coefficients for back faced strain relationship.

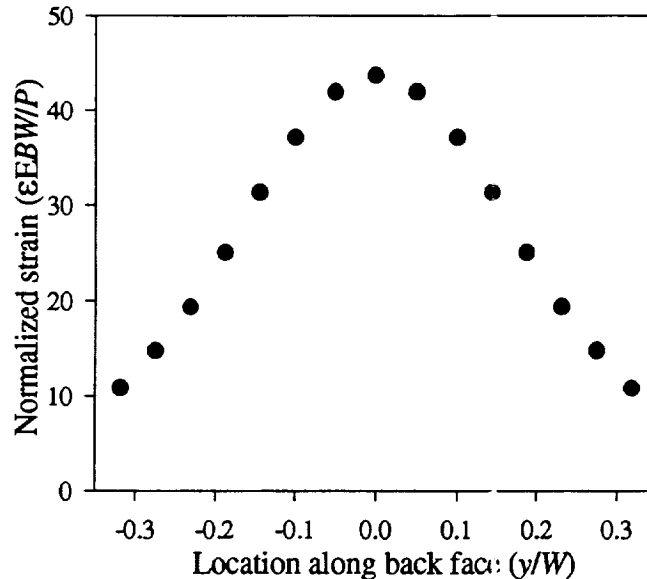
$N_0$	-0.07978
$N_1$	0.33982
$N_2$	-0.64978
$N_3$	0.53227
$N_4$	-0.21704
$N_5$	0.03154

### Effect of Finite Sized Strain Gages

The coefficients given in Table 2 are based on strain at the centerline ( $y = 0$ ) of the back face of the compact tension specimen. In practice, finite sized strain gages measure strains over a finite gage length. This might cause the measured strain to be somewhat less than that occurring at the center line of the specimen. The variation in strain along the back face of a specimen with  $a/W = 0.7$  is shown in Figure 6. These strains were calculated from the first derivative of the polynomial fitted to the nodal displacements. The percent change in strains that are measured with finite-sized gage lengths, compared to the value of strain at  $y = 0$  are shown in Figure 7 for three different values of  $a/W$ . Although there is relatively little effect of the gage length on measured strain when the gage length is on the order of  $0.1W$ , larger strain gages can affect the measured value of strain, especially as  $a/W$  increases.

### Summary

A numerically generated expression for crack length in a compact tension specimen that is based on back face strain is presented. This expression is given in a form similar to that previously published for a back faced strain expression for the extended compact tension specimen. The numerically generated expression for the compact tension specimen presented in this paper is bounded by the two experimentally generated expressions that were previously presented in the literature.

Figure 6. Variation of strain along back face of compact tension specimen with  $a/W = 0.7$ .

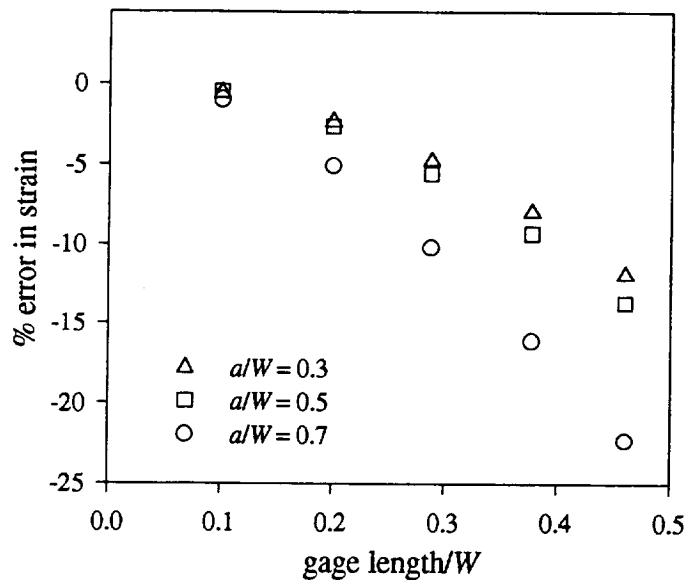


Figure 7. Effect of finite-sized strain gages on measured strain for three different  $a/W$  values.

## Acknowledgments

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## Abstract

*A numerically generated expression to determine crack length in a compact tension specimen from back face strain compliance is presented. The numerically generated back face strain expression is bounded by two experimentally determined expressions previously published in the literature. Additionally, stress intensity factor and crack mouth opening expressions are determined. These expressions agree well with previously published results.*

## Introduction

Compliance-based techniques for crack length measurement are commonly used in automated fatigue crack growth rate testing. Common techniques for compliance measurement are a crack mouth mounted clip gage [1] or a back face mounted strain gage [2]. The compact tension specimen, shown in Figure 1, is a standard specimen geometry for fatigue crack growth rate testing [3]. The crack mouth opening [4] and stress intensity factor [5,6] expressions for this configuration are well established [3]. In contrast, there is some disagreement in the literature regarding experimentally generated back faced strain expressions for the compact tension specimen [2,7].

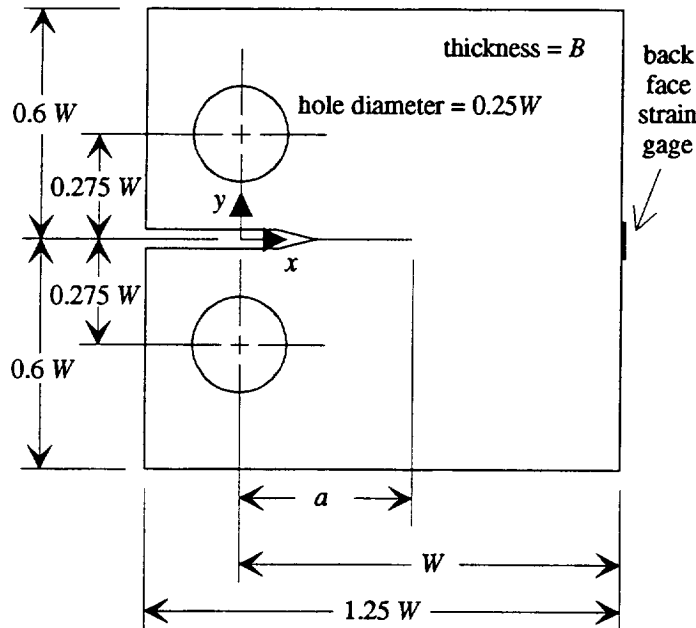


Figure 1. Configuration of compact tension specimen for fatigue and fracture mechanics-based testing.

The purpose of this paper is to present a numerically generated back-faced strain expression for crack length in compact tension specimens and to resolve the differences between previously published, experimentally derived expressions. The effect of finite sized strain gages, necessary in laboratory procedures, is investigated. The well-established relations for crack mouth opening displacement and stress intensity factor are used to evaluate the accuracy of the present analyses.

## Numerical Procedure

The finite element code FRANC2D [8] was used to perform two-dimensional linear-elastic stress analyses on compact tension specimen configurations. Crack length to width ratios ( $a/W$ ) from 0.1 to 0.9 were considered. The FRANC2D is an ideal tool for these simulations because the topology-based data structure allows crack extension with only local remeshing. This feature allows many different values of  $a/W$  to be investigated with a minimum of preprocessing time.

For each analysis, a rosette of singular quarter point elements was inserted surrounding the crack-tip. Each side of the singular elements was approximately  $0.0325W$ . The remainder of the body was meshed with rectangular or triangular quadratic elements. A typical mesh, for  $a/W = 0.5$ , is shown in the (grossly exaggerated) deformed state in Figure 2. A more refined mesh for  $a/W = 0.5$  was also generated and analyzed. The refined mesh had approximately 4 times as many elements as the typical mesh. The strains and stress intensity factors resulting from the typical mesh, shown in Figure 2, and the refined mesh were practically identical. This good agreement validates the typical mesh refinement used for these analyses.

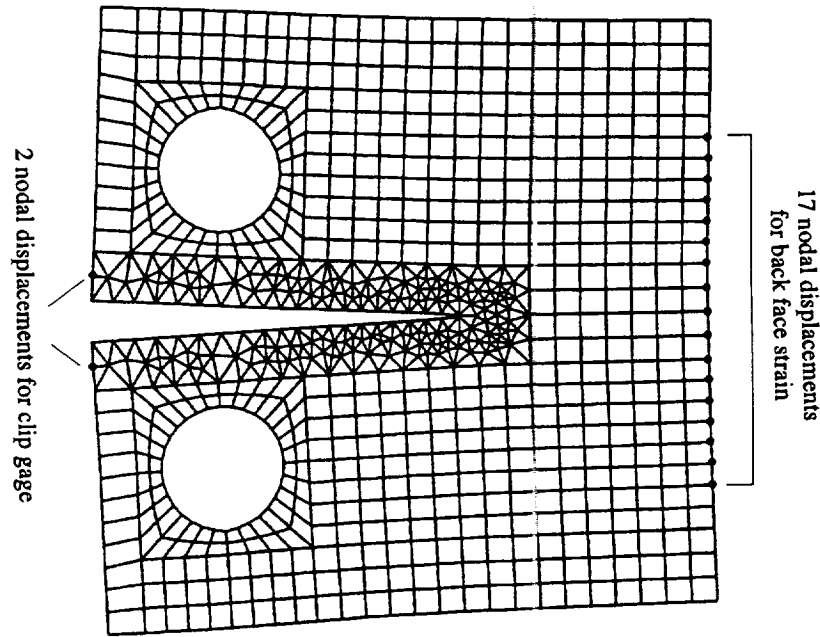


Figure 2. Finite element mesh for analysis of compact tension specimen with  $a/W = 0.5$ .

Nineteen salient nodes of the finite element model were identified. Seventeen of these nodes were located along the back face surface of the specimen (Figure 2), while two were located at the crack mouth (Figure 2). The nodal displacements at these nineteen nodes are used in the present analysis to determine simulated back-face-mounted strain gage and crack-mouth-mounted clip gage responses, respectively. The simulated clip gage response was determined by the relative displacement,  $v$ , between the two designated nodes. A polynomial was fitted to the y-direction displacements of the seventeen designated nodes along the back face. A 5th order polynomial produced a good fit of the displacements for  $a/W \leq 0.6$ . Higher order polynomials were required to obtain a good fit to these displacements for  $a/W = 0.7$  and  $0.8$  (7th order) and  $a/W = 0.9$  (9th order). The first derivative with respect to  $y$  of each polynomial expression yields the strain in the  $y$ -direction ( $\epsilon_{yy}$ ) along the back face of the specimen for the given  $a/W$ .

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